THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMAT5540 Advanced Geometry 2016-2017
Suggested Solution to Assignment 1

1. Length of the curve $= \displaystyle\int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \sqrt{17} + \frac{1}{4}\ln(4+\sqrt{17})$

2. Length of the curve $= \displaystyle\int_0^3 \sqrt{1+[f'(x)]^2}\,dt = 2\sqrt{65} + \frac{1}{4}\ln(8+\sqrt{65}) - \frac{\sqrt{5}}{2} - \frac{1}{4}\ln(2+\sqrt{5})$

3. To prove the statement, it suffices to prove that if the relation $\sim$ is reflexive, the relation $\sim$ is symmetric and transitive if and only if the relation $\sim$ satisfies the second condition.

   "$\Rightarrow$" Assume that the relation $\sim$ is reflexive, the relation $\sim$ is symmetric and transitive.
   Suppose that $a \sim b$ and $a \sim c$.
   By symmetry, we have $b \sim a$. Also, by transitivity, $b \sim a$ and $a \sim c$ implies $b \sim c$.

   "$\Leftarrow$" Assume that the relation $\sim$ satisfies the second condition.
   Suppose that $a \sim b$.
   Together with $a \sim a$, we have $a \sim b$ and $a \sim a$, it implies $b \sim a$ by the second condition. Therefore the relation $\sim$ is symmetric.
   Suppose that $a \sim b$ and $b \sim c$.
   By symmetry as shown above, we have $b \sim a$. Then $b \sim a$ and $b \sim c$ implies $a \sim c$ by the second condition. Therefore the relation $\sim$ is transitive.

4. (a) The statement is true. Let A and B are distinct points, consider the perpendicular bisector of the line segment AB, then A and B lie on the opposite side of it.

   (b) The statement is false. If A, B and C are three distinct points that are collinear, then there exists no circle passing throught all of them.

5. (a) Let a, b and c be integers.
   Since $a - a = 0$ which is divisible by n, $a \sim a$.
   Suppose that $a \sim b$, then $b - a = np$ for some integer p.
   Then $a - b = -np = n(-p)$ which is divisible by n, so $b \sim a$.
   Suppose that $a \sim b$ and $b \sim c$, then $b - a = np$ and $c - b = nq$ for some integers p and q.
   Then $c - a = (c-b) + (b-a) = n(p+q)$. $p+q$ is an integer, so $c - a$ is divisible by n and $c \sim a$.
   As a result, $\sim$ is an equivalence relation.

   (b) $\mathbb{Z}_n := \mathbb{Z}/\sim = \{[0], [1], \cdots, [n-1]\}$.

   (c) It suffices to show that if $a \sim a'$ and $b \sim b'$ then $a + b \sim a' + b'$.
   Suppose that $a' - a = np$ and $b' - b = nq$ for some integers p and q.
   Then $(a' + b') - (a + b) = n(p+q)$. $p+q$ is an integer, so $(a'+b') - (a+b)$ is divisible by n and $a + b \sim a' + b'$.

(d) $[21] + [35] = [21 + 35] = [56] = [2]$.

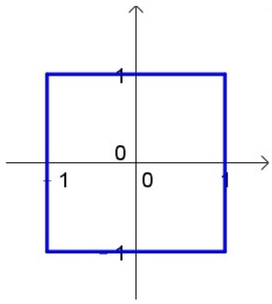
Alternative method: $[21] + [35] = [3] + [5] = [8] = [2]$.

6. (a) $\phi(s) = \sqrt{(10-2)^2 + (11-3)^2} = 8\sqrt{2}$.

(b) It follows from the fact that equality of real numbers is an equivalence relation.

7. (a) $d((1,-2),(-3,4)) = \max\{|-3-1|, |4-(-2)|\} = \max\{4,6\} = 6$

(b)



(c) $d((1,-2),(-3,4)) = |-3-1| + |4-(-2)| = 4+6 = 10$